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Review of Computational Aeroacoustics in Propulsion Systems

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Recent efforts focusing on computational aeroacoustics in propulsion systems are reviewed. Difficulties associated with a finite difference solution of the time-dependent governing equations and boundary treatments are briefly discussed. Success and limitations of the large-eddy-simulation (LES) approach in which the sound source and the radiation field are simultaneously obtained are presented. It is suggested that LES be limited to the near field, and that other techniques be used to extend the near field to the far field. Several extension techniques are given. Approximate techniques for fast prediction of the source regime are reviewed. This is followed by discussing the coupling between the engine internal flow and the jet plume noise.

I. Introduction

THE growing interest in computational aeroacoustics is largely a result of efforts to develop high-speed civil transport. The success of this new technology is contingent upon reducing its jet exhaust noise.[1] Despite recent efforts to introduce quieter aircraft, it is expected that noise will increase as a barrier to air transportation.[2–4] This is attributed to the introduction of faster aircraft, tripling of air traffic in the next 20 years, and increased urbanization around airports. The present paper reviews efforts directed at first-principles prediction of jet exhaust noise and related engines' internal flow.

Noise is generated by the time-dependent turbulent fluctuations in the near field of the jet. These fluctuations propagate to the far field producing the radiated sound. The major difficulty in predicting jet noise is obtaining the instantaneous turbulent fluctuations in the source regime. Reynolds-averaged equations can give the mean intensity of turbulence, but not the fluctuations' time-history. Thus, Reynolds-averaged equations cannot directly provide the sound source.

Apart from empirically relating noise to mean-flow parameters,[5] early attempts for the prediction of jet noise were based on modeling the time-dependent sound source in the near field by semi-analytical solutions. Acoustic analogy or asymptotic methods are then used to calculate the noise field associated with this source solution.[6–12] Recently, the direction for jet-noise prediction has shifted toward the application of computational aeroacoustics (CAA) to calculate the jet's unsteady flow and its radiated sound.

We review herein first-principles approaches for the prediction of noise in propulsion systems, with focus on jet noise and internal engine noise. In Sec. II, the numerical issues associated with CAA are discussed. In Sec. III, the large-eddy-simulations (LES) approach will be presented. For the three-dimensional case, current computer capabilities make direct LES of near and far field prohibitive. It is more practical for the three-dimensional case to restrict LES to the near field, and to use extension techniques to obtain the far-field sound. Such extension techniques will be discussed in Sec. IV. Approximate techniques to obtain fast prediction of the source region are discussed in Sec. V. In Sec. VI, we discuss engines' internal flow, which precedes the jet plume exit and plays a key role in controlling far-field noise.

II. Numerical Issues

To appreciate the numerical difficulties associated with jet-noise simulation, let us first briefly examine the physics involved.[13] The streamwise development of the jet can be split into three regimes. In the potential core regime, the shear layer, formed at the nozzle lip, spreads and reaches the centerline of the jet, marking the end of the potential core. The mean-flow centerline velocity is constant within this core. This is followed by the transitional regime until the fully developed regime is reached. A computation domain needs to extend $60D$ downstream before the centerline velocity has considerably decayed. Here, D is the nozzle diameter. In measuring the acoustic field, the microphone is usually placed at a circle centered at the jet exit of at least a $40D$ radius. Thus, the computational domain must extend radially to about $40D$. Though the mean flow of the jet may be axisymmetric, its unsteady structure is three dimensional, and, in supersonic jets, the first helical mode could dominate over the axisymmetric mode. Three distinct scales can be identified. In the acoustic field, the disturbance scales with the acoustic wavelength. In the jet flow, the structure can be classified into two. One scales with the nozzle diameter and is sometimes called the *jet column mode*.[14] The other, sometimes called the *shear-layer inability mode*, scales with the initial momentum thickness of the shear layer, which is about 3% of the nozzle diameter. The relevant frequencies can be as high as $St = 10$, where $St = fD/U_j$, f is the frequency in Hz, and U_j is the jet exit velocity. In supersonic jets, besides mixing noise, shock-cell structure is formed and interacts with the unsteady disturbances, producing broadband shock-associated noise and the screech tones. To resolve all of these scales in such an extended computational domain, one must use a high-order scheme that requires minimum grid points per wavelength to accurately capture the disturbance field.

As the disturbances in the flowfield propagate to the acoustic field, they become less than 10^{-4} of the mean flow. If both the mean flow and the disturbances are to be captured accurately, the numerical error must be much less than that. Furthermore, because of the long propagation length, numerical dispersion and dissipation errors accumulate. As such, dissipation and dispersion error should be minimized.

The computed solution could also be contaminated by spurious modes. These modes arise from the boundary conditions imposed at the computational boundary that mimics the physics only at infinity. It could also arise from the fact that the dispersion relation of the discretized equations differs from that of the differential one, or from sudden changes and stretching of the grid.[15]

In the following text, we briefly discuss finite difference schemes and boundary treatments relevant to CAA of jet noise. For extensive reviews on these issues the reader is referred to Refs. 16–19.

A. Algorithms for Computational Aeroacoustics

Two workshops were held to address numerical issues pertaining to CAA.[20,21] A number of benchmark problems were solved using various algorithms under controlled parameters. Several schemes were found to handle linear acoustic propagation quite well.[22–28] Few schemes performed well for nonlinear problems. Problems

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involving shock–acoustic interactions or nonuniform curvilinear grids represent additional difficulties. Capturing shocks accurately usually requires dissipation, which could dampen the acoustic waves.[29,30] Nonuniform curvilinear meshes could reduce the order of accuracy.[31,32] Three discretization schemes, which have been successfully used for CAA of jet-noise problems, will be briefly given herein. Consider the one-dimensional equation

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \frac{\partial F}{\partial x} = 0 \tag{1}$$

where $\boldsymbol{Q}$ is the unknown vector and F is the flux in the x direction. The MacCormack 2–4 scheme is fourth-order-accurate in space, second-order-accurate in time, which is an extension of the classical MacCormack[33] scheme owing to Gotleib and Turkel.[34] The operator is split into one-dimensional operators; each operator consists of a predictor and corrector steps using one-sided differences as follows:

$$\begin{aligned} Q_i^{n+\frac{1}{2}} &= Q_i^n - (\Delta t/6\Delta x)(7F_i - 8F_{i-1} + F_{i-2})^n \\ Q_i^{n+1} &= \tfrac{1}{2}\left[Q_i^{n+\frac{1}{2}} + Q_i^n + (\Delta t/6\Delta x)(7F_i - 8F_{i+1} + F_{i+2})^{n+\frac{1}{2}}\right] \end{aligned} \tag{2}$$

and similarly for the other directions. Because the operator is split, boundary treatment can be applied with relative ease.

The compact scheme of Lele[35] is quite attractive, and can be written in the form

$$\alpha D_{i-1} + (1-2\alpha)D_i + \alpha D_{i+1} = (1/\Delta x)(aF_{i-1} - aF_{i+1}) \tag{3}$$

where D is the operator and the coefficients a and α can be optimized to achieve the required degree of accuracy. The scheme has successfully been used to simulate several CAA problems.[17] However, it is less friendly to boundary treatment, and often an exit zone or sponge layer needs to be imposed, such that spurious modes generated at the boundaries are gradually dampened. Hixon[36] has attempted to overcome this difficulty by splitting the operator D into forward D^F and backward D^B, and writing the discretized equations in the form

$$\begin{aligned} &cD_{i-1}^F + [1-(a+c)]D_i^F + aD_{i+1}^F \\ &\quad = (1/\Delta x)[-(1-b)F_{i-1} - (2b-1)F_i + bF_{i+1}] \\ &aD_{i-1}^B + [1-(a+c)]D_i^B + cD_{i+1}^B \\ &\quad = (1/\Delta x)[-bF_{i-1} + (2b-1)F_i + (1-b)F_{i+1}] \end{aligned} \tag{4}$$

The stencil size for a sixth-order compact is reduced from five points to three points as a result of splitting, which helps the boundary stencil specification. Other discussion of boundary treatments for compact schemes can be found in Ref. 37.

The dispersion-relation-preserving (DRP) scheme of Tam and Webb[38] can be written as

$$Q_{l,m}^{(n+1)} = Q_{l,m}^{(n)} + \Delta t \sum_{j=0}^{3} b_j K_{l,m}^{(n-j)} \tag{5}$$

where $K_{l,m}^{(n)}$ is a function of the flux and is given, along with the coefficients, in Ref. 38. The coefficients of the scheme are chosen to represent more accurately the wave components over a wide range. For simple benchmark problems, the DRP scheme requires fewer grid points per wavelength than in the fourth-order MacCormack case, but with more difficulty in boundary treatment. However, considerable progress has been made by Tam and Webb in developing boundary treatments suitable for the DRP scheme.

B. Boundary Treatment for CAA

The computational domain is usually finite and numerical boundary treatments need to be applied at the boundaries to depict conditions at infinity. This could generate spurious modes that propagate to the interior domain and render the computed solution entirely unacceptable. Unsteady boundary treatment represents serious difficulty for CAA, but considerable progress is made in this regard. There are several proposals for boundary treatments based on various ideas. In Refs. 39–41, characteristic-based boundary conditions are given. Absorbing boundary treatment is given in Refs. 42–44. Sponge, buffer, or exit zone treatments are given in Refs. 45–47, whereas filtering is proposed in Ref. 48. The perfectly matched layers (PML) technique used in electromagnetics[49,50] has been extended to computational aeroacoustics in Refs. 51 and 52. For an illuminating description of boundary treatment requirements and a state-of-the-art review of available methods, the reader is referred to Tam.[53] Comparisons between the performance of some of these methods are given in Refs. 54–57. We present boundary treatments that NASA Lewis Research Center's group has found to be the most successful in simulating jet-noise-related problems.

1. *Wall Boundary Conditions*

On solid surfaces, the usual no-slip, no-through-flow boundary condition can be applied. But to maintain the accuracy of the scheme, the minimum ghost points approach of Refs. 58 and 59 is adopted. This has proven to be a successful treatment when coupled with bias discretization.

2. *Outflow Boundary Treatment*

Asymptotic analysis of the linearized Euler equations (LEE) for large distances has been performed in several references.[38,60–62] For an outgoing-wave solution, the boundary condition on the pressure can be stated as

$$\frac{1}{\mho}\frac{\partial p'}{\partial t} + \frac{\partial p'}{\partial R} + \frac{p'}{R} = 0$$

where

$$\begin{aligned} \mho &= c_0\left\{(x/R)M + \lfloor 1 - (r^2/R^2)M^2 \rfloor^{\frac{1}{2}}\right\} \\ R &= \sqrt{x^2 + r^2} \end{aligned} \tag{6}$$

Here, p is the pressure, and prime denotes the disturbance. The speed of sound is c_0, and M is the local Mach number. However, the asymptotic analysis in Ref. 38 shows that the velocity and density disturbances are composed of both acoustic and flow disturbances. Thus, the governing equations, except the pressure (energy) equation, are not modified, but are one-sided differenced at the boundaries to account for the presence of flow disturbances. The pressure equation is replaced by its asymptotic approximation that ensures outgoing waves. The treatment produces acceptable solution as long as the mean flow at the outflow is not too steep.

3. *Radiation Boundary Condition*

At computational boundaries where acoustic radiation dominates, the conventional acoustic radiation condition based on the asymptotic analysis of the wave equation applies. Namely,

$$\{u', v', p'\}_t = -\mho\left[\frac{\partial}{\partial R} + \frac{1}{R}\right]\{u', v', p'\} \tag{7}$$

Here, p is the pressure; u and v are the velocities in the axial direction x, and the radial direction r, respectively; and prime denotes the disturbance quantity. This treatment was found to be quite robust, producing no reflection at radiation boundaries.

4. *Inflow Treatments*

For supersonic inflow, all characteristics are incoming, and inflow treatment poses no difficulty. However, for subsonic inflow, reflections occur that could lead to unstable solution. Unlike the cases of walls, radiation, and outflow, none of the existing treatments for subsonic inflow stand out as completely satisfactory for jet simulations. But Thompson's,[39,40] Giles's,[63] and Tam et al.'s[64] ideas were found to produce a stable solution marginally free from reflections. Each of these methods works fine for some situations, but not for all situations.

In Thompson's analysis,[39] the nonlinear Euler equations for one-dimensional flow are decomposed into wave modes of definite velocity. The acoustic waves propagate at sound speed relative to the mean flow. The vorticity and the entropy waves are frozen patterns convected downstream by the mean flow. The outward-propagating waves are defined entirely by the state of the variables within the computational domain. The behavior of incoming waves is specified by data external to and on the boundary. For nonreflecting inflow, the amplitude of the inward propagating wave is set to zero. For extension to the two-dimensional case, the transverse terms are taken as a passive source term. The method is acceptable if the incoming flow is nearly one dimensional perpendicular to the inflow surface.

Giles[63] derived boundary conditions based on Fourier analysis of the two dimensional LEE with constant coefficients. The dispersion relation is expanded in a Taylor series around the one-dimensional solution to obtain boundary conditions for various degrees of approximation. The outgoing characteristic is obtained from the interior numerical solution. The incoming characteristics are modified to prevent reflection.

Tam et al.'s[64] asymptotic analysis of LEE indicates that for inflow treatment, the flux can be updated as in the radiation condition, but the effect of imposed inflow disturbances is accounted for. This proposal produced nonreflecting stable solution when used in Ref. 57 for inflow treatment of a subsonic jet.

III. Large-Eddy Simulations

The full, time-dependent, compressible Navier–Stokes equations govern the sound generation and propagation. Flow and acoustic fluctuations can be obtained via accurate, direct numerical simulations (DNS) of these equations. For a thorough review of progress in this direction, the reader is referred to Lele[17] and Morris et al.[65] Because of current computer capabilities, DNS is limited to low-Reynolds-number simulation of simple configurations. Such as, sound scattering by vortex,[66] sound generation by vortex pairing,[67] and radiation from shear layers.[68,69]

In the LES approach, the full compressible Navier–Stokes equations are solved, as in DNS, for both the flow and acoustic fluctuations. But for the technologically important high Reynolds number flows, not all of the scales of motion are resolved numerically. The effect of the unresolved scales on the resolved ones is accounted for via modeling. In LES, the flowfield is decomposed into filtered and residual fields

$$f = \bar{f} + f'' \tag{8}$$

where an overbar denotes the filtered (resolved) field and a double prime denotes the unresolved (subgrid) field. Upon substituting this splitting into the full Navier–Stokes equation, the filtered compressible Navier–Stokes equations in cylindrical coordinates are obtained.[70] The LES equations for CAA are essentially the same as those in conventional LES.[71,72] However, in conventional LES, the objective is accurate prediction of the mean flow and turbulence intensity. But in CAA, the objective is accurate prediction of the flow and acoustic fluctuations. To capture the acoustically relevant scales, the resolution requirements may be higher than in conventional LES. Furthermore, because of the fluctuating nature of the solution, high-order differencing scheme and boundary treatments are needed to reduce dissipation and dispersion errors, and one has to ensure that the captured fluctuations are not contaminated by spurious modes. In the outer acoustic field, the resolved scales are the large-wavelength acoustic waves, and, as such, the terminology large-scale simulations (LSS) may be more appropriate than LES.

The hypothesis used in introducing LES for CAA in Ref. 73 is that the large-scales are more efficient than the smaller scales in radiating noise. This hypothesis is supported by numerous observations.[74–78] A recent study by Tam et al.[79] indicates that jet mixing noise can be attributed to two components: a large-scale organized one and a fine-scale random one. The radiation of the latter is not always negligible. In using LES for CAA, the resolved scales could encompass the large-scale organized component and some fine-grained random component, depending on the resolution. Thus, one needs to address the question of what scales are accurately resolved by LES and their relation to the acoustically relevant scales.

In LES, the effect of the unresolved scales on the resolved ones need to be modeled. The importance of this issue depends on the range of resolved scales. As pointed out earlier, in CAA one needs to accurately resolve a certain range of acoustically relevant scales. This may require finer resolution than that required for accurate prediction of the mean flow and turbulence intensity. With higher resolution, the residual scales should diminish and the effect of subgrid modeling becomes less important. In Ref. 73, a compressible version of the conventional Smagornisky's model[80] is used for LES simulation of the source region of $M = 1.4$ supersonic jet. Nonlinearity caused generation of harmonics, subharmonics, and sum and difference modes of the initially dominant frequency. The captured structure was found to be wavelike, resembling nonlinear instability waves. In this case, two-way energy transfer between the large and smaller scales exists.[81,82] This, and other physics, are not accounted for in Smagornisky's model because it is on eddy-viscosity type. Thus, although Smagornisky's model may be adequate for fine grids, it cannot be used with confidence for coarse grids. Further studies are needed to address the question of subgrid modeling for CAA.

Simulations of the near and far field associated with axisymmetric large-scale structures in a supersonic round jet are given in Refs. 83 and 84. The jet is perfectly expanded at Mach number $M = 2.1$ and $Re = 7 \times 10^4$. Both the near flowfield and the radiated acoustic field are simultaneously captured. The development of the initial region of the jet, where most of the noise comes from, is dependent on the inflow conditions.[85,86] As such, one needs to specify the type of inflow disturbance when calculating the radiated noise field.

Figure 1 shows photographs of the pressure field at four cases of inflow disturbances. The top two figures are initially single frequency, corresponding to $St = 0.2$, and the initial disturbance level is small or large, $\varepsilon = 0.001$ and $\varepsilon = 0.04$. In the bottom two figures, $\varepsilon = 0.04$, but the initial frequency is either bimodal corresponding to $St = 0.2$ and 0.4, or completely random. Focusing on the source regime around $r/D = 0.5$, Fig. 1 shows that for the small initial amplitude, the disturbance's growth is exponential in accordance with the linear stability theory and no saturation is reached within the computational domain. As such, the looped-shaped radiation pattern is not observed. For the large input disturbances, corresponding

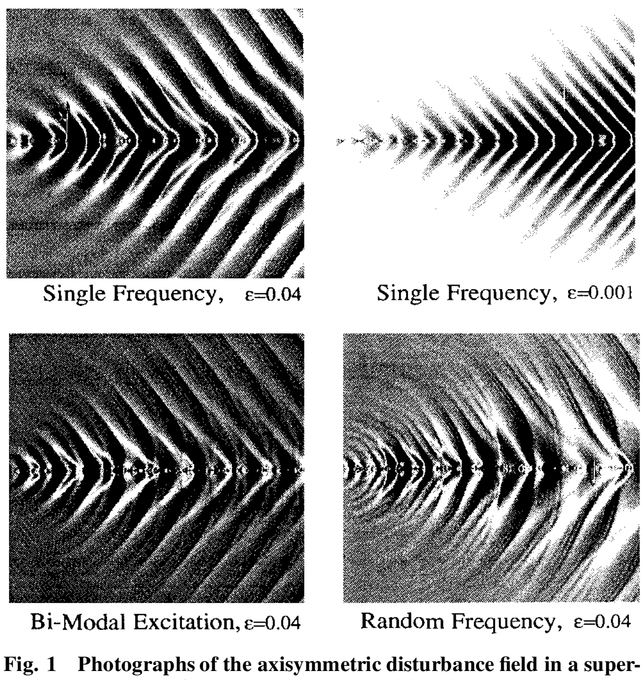

Fig. 1 Photographs of the axisymmetric disturbance field in a supersonic jet at various frequency modes and initial disturbance levels ε.

to $\varepsilon = 0.04$, the disturbances initially grow exponentially as in the linear stability theory, but this is followed by nonlinear saturation of the disturbances amplitude. As the initial amplitude or frequency modes increase, the peak moves upstream. The acoustic waves radiate forward and seem as if they originate from the streamwise locations of the source peak. For cases where the source saturates, a looped-shaped radiation pattern is obtained. For inflow disturbances with a single frequency, the peak of the radiation pattern is well defined. Two-frequency inflow disturbances are associated with two peak radiation angles. In the random inflow disturbance case, although initially there is no dominant frequency, a *preferred* frequency develops corresponding to $St = 0.2$.

The wavy nature of the solution is also apparent in Fig. 1, even for the initial random inflow disturbances. Figure 1 shows full computational domain, and very few reflections from the boundaries can be seen. As such, the success of the present boundary treatment allows one to efficiently use the computer facilities without having to exclude an exit region of the computed solution.

Three-dimensional LES of jet noise, including the source regime and the acoustic field, requires prohibitive computer time. Shih et al.[87] used LES to calculate the three-dimensional near field of a supersonic jet, with special attention given to centerline treatment. Photographs of the flowfield and the surfaces of constant pressure and constant kinetic energy in a supersonic jet indicated that the helical nature of the structure dominates. Direct extension to the farfield is computationally expensive. Other techniques are needed to obtain the far-field sound associated with the LES of the three-dimensional near field.

IV. Near-Field to Far-Field Extension Techniques

As pointed out in the previous text, direct LES of the three-dimensional acoustic field may not be feasible because of current computer capabilities. The alternative is the zonal approach,[88] in which LES is limited to the near-field source regime, and other less CPU intensive approaches are used for extension to the acoustic field.

Early attempts for jet-noise prediction relied heavily on Lighthill's theory,[89] in which the far-field noise is given in terms of a volume integrand of stress tensor representing the sound source. For subsonic jet noise, Lighthill's prediction gave results consistent with observation, but some discrepancies were also reported.[90–94] This discrepancy could be attributed to the fact that the mean flow–acoustic interactions are not explicitly accounted for in Lighthill's theory, or to inaccurate specification of the sound source. Application of Lighthill's theory in connection with LES is given in Ref. 73, where LES of a supersonic jet is used to calculate Lighthill's integrand. The azimuthal integration was analytically performed in terms of Bessel function. The Lighthill's integrand was found wave-like oscillating in the x direction. The far-field sound is the net cancellation upon performing the volume integral. The source may not completely vanish within the computational domain, which is usually the case for supersonic jets. For such noncompact sources, Lighthill's volume integral may not converge within the computational domain. This leads to an arbitrary noise level, depending on the cutoff location. The alternative to acoustic analogy is to use extension techniques, as outlined next, to extend the inner, source solution to the acoustic radiation field.

A. Kirchhoff's Surface-Integral Formulation

The Kirchhoff's formulation for the acoustic field associated with a sound source in a uniform stream is given in Morino.[95] In this approach, the sound pressure is given in terms of a surface integral, including information over a surface that encloses all the sound sources. Kirchhoff's formulation is extensively used in prediction of turbomachinery noise.[96–99] Application of Kirchhoff's method to jet noise prediction is given in Refs. 100–103. LES or DNS is used to calculate the field inside a cylindrical surface enclosing the sound source. The numerically obtained pressure and its normal derivative on this cylindrical surface are then used in Kirchhoff's surface integral to obtain the acoustic radiation. While the classical Kirchhoff's method does produce qualitatively acceptable results for jet-noise prediction, two main difficulties of applying it to the jet-noise problem were highlighted in Ref. 100: namely, matching, and the need to numerically calculate the normal derivative of the pressure. Regarding matching, we note that Kirchhoff's solution is that of the wave equation. The input surface pressure must also be a solution of the wave equation. The proper radiation boundary condition must, therefore, be used for the source domain computation. But boundary conditions mimic the situation only at infinity, and there is no guarantee that the solution at the boundaries of the source's finite domain is a solution of the wave equation. Thus, unless careful attention is given to boundary treatments at the matching surface, Kirchhoff's solution might depend on the location of the matching surface.

B. No-Derivative Surface-Integral Formulation

The second difficulty associated with Kirchhoff's solution is that not only the pressure signal is needed at Kirchhoff's surface, but also its normal derivative. Noting that the computational mesh is usually coarse near the outer boundaries, the pressure-differencing process introduces errors. As such, for two-dimensional airfoils Atassi et al.[104] developed a surface-integral formulation that does not require the normal derivative of the pressure. A similar approach for jet-noise prediction was developed in Ref. 105, which requires only the pressure signal on a cylindrical surface surrounding the sound source. The solution for the acoustic field in terms of the frequency and azimuthal Fourier components of the surface pressure is given by

$$p(x_o, r_o, \chi_o, \omega) = \frac{1}{2\pi}\sum_{n=0}^{n=\infty}\cos n\chi_o \times \int p_n(x,\omega)\int_{-\infty}^{\infty} e^{i(x-x_o)K_x}\frac{H_n(qr_o)}{H_n(qa)}\,\mathrm{d}K_x\,\mathrm{d}x \tag{9}$$

where the subscript o denotes the observer, subscript n denotes the azimuthal mode, and a is the radius of the cylindrical surface encompassing the source, and

$$q = \sqrt{(\omega/c)^2 - K_x^2} \tag{10}$$

C. Finite Difference Solution of the Linearized Euler Equations

The LEE with nonuniform mean flow is more accurate than the convective wave equation in describing the disturbance field outside the inner field. The LEE can be obtained by neglecting viscosity in Navier–Stokes equations. The flowfield is then split into a mean and a time-dependent components. Upon substituting this splitting into Euler equations and linearizing around the mean flow, the LEE are obtained.[70] Finite difference can be used to solve for LEE, but the accuracy of the finite difference solution must be assessed. Because nonlinearity and viscosity are absent from LEE, spurious modes resulting from differencing and boundary treatment are not likely to be dampened. The computed disturbance field can easily be contaminated by modes other than the physical ones. Several criteria, beside grid independence, must be considered to ensure the accuracy of the computed solution.[106] Namely, 1) the spectra should not exhibit frequency-components other than the input modes, 2) vorticity should vanish in the acoustic field, 3) the acoustic pressure decays as $1/r$, and 4) the global instantaneous field should exhibit no boundary reflections.

Troutt and McLaughlin[107] measured the disturbance field associated with low Reynolds number $M = 2.1$ supersonic jet excited with axisymmetric disturbances. The angular directivity of the sound pressure level for a supersonic jet calculated via LEE is shown in Fig. 2 along with the corresponding measurements of Troutt and McLaughlin. In the experiment the jet was excited by a single-frequency axisymmetric component, but the first-helical mode was found to develop downstream and reach amplitudes comparable to that of the axisymmetric mode. Figure 2 shows the results of both the axisymmetric and helical modes. Because this is a linear solution, the calculated contribution of the two modes can be superimposed to give the total noise field. Tam and Burton[108] obtained an asymptotic solution of the acoustic radiation of a single instability wave

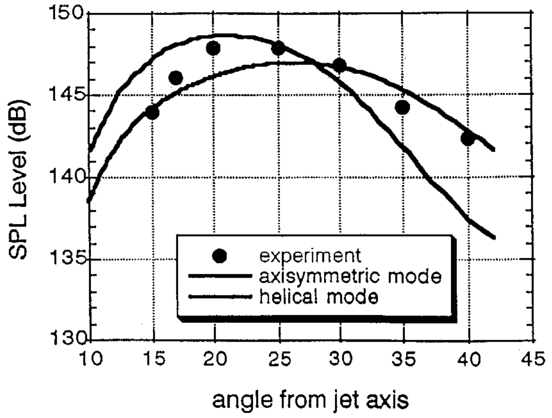

Fig. 2 Directivity of jet noise associated with the axisymmetric and the helical modes in a supersonic jet compared with Troutt and McLaughlin's data.[107]

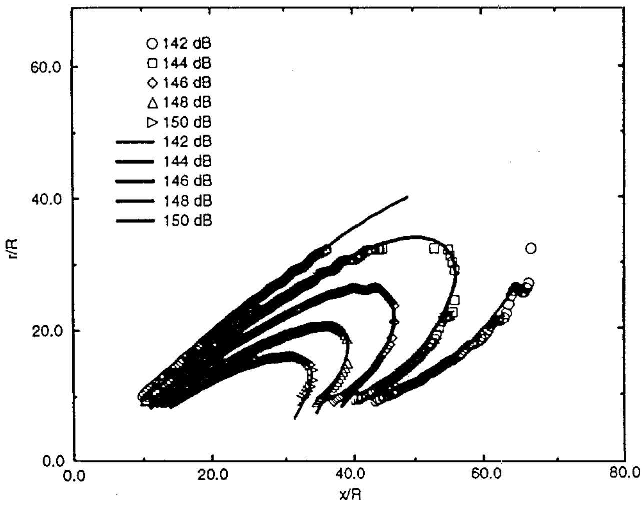

Fig. 3 Predicted contours of the sound pressure level using LEE in comparison with the asymptotic solution for axisymmetric instability wave in a supersonic jet. Symbols = current numerical calculation. Lines = analytical solution.

in a supersonic jet. The LEE solution encompasses that of the linear stability solution. The calculated LEE solution was found in Ref. 106 to be in close agreement with the corresponding stability wave solution of Ref. 108, as shown in Fig. 3.

D. Evaluation of Extension Techniques

Shih et al.[109] performed a comprehensive comparison of various extension methodologies for the far-field jet noise. The computation consists of two parts. LES is used to obtain the nonlinear near field enclosing the sound source for an $M = 2.1$ supersonic jet. The LES results are then used to obtain the sound field using other extension techniques. Figure 4 shows the calculated sound field directivity at a circle of radius $48D$, where D is the nozzle diameter. Direct prediction by LES is compared in the figure to the predictions of Lighthill's LEE, Kirchhoff surface-integral formulation, the no-derivative surface-integral formulation, and the modified Kirchhoff of Ref. 98. A considerable shift in Lighthill's prediction is apparent; but all the other methods are practically as accurate as LES; and a considerable reduction in CPU time is achieved.

Thus, for extending LES simulation of near field to the far field, LEE and the two surface integral formulations can produce accurate results. LEE is more general than a surface-integral or asymptotic approach in that variation in the mean flow is considered. Also, unlike solutions based on linear stability assumptions, LEE solution includes all disturbance modes besides those related to the linear stability. However, for very far-field computation, LEE becomes

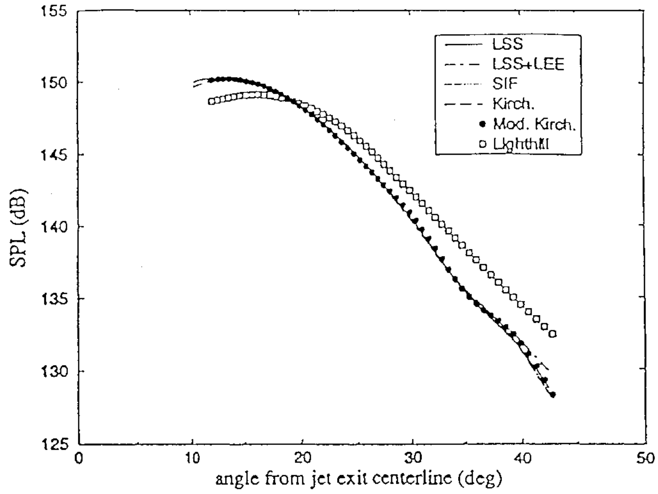

Fig. 4 Directivity of jet noise based on various extension techniques in comparison with direct LES.

CPU intensive compared to SIF. The KSIF depends on the accuracy of the normal derivative of the pressure. The smaller Δr, the more accurate the solution will be. The NDSIF does not suffer from this inaccuracy.

V. Approximations to the Sound Source

Limiting LES to the source regime and using other faster techniques for extension to the far field considerably reduces computational time. However, approximations to the source regime itself might still be desirable to speedup three-dimensional computations. Some of these methodologies are discussed next.

A. Linear Stability Theory

The linear stability theory has been used considerably to approximate the sound source in jets and shear layers.[110–114] The basic hypothesis is that the unsteady fluctuations in the noise-producing initial region of the shear-layer or jet are dominated by coherent, large-scale structure that resembles instability waves. The results qualitatively agree with observation. Nonlinear effects are not accounted for, and only instability modes can be considered. Other techniques are needed to obtain the acoustic field such as acoustic analogy or matched asymptotic expansion.[108]

B. Linearized Euler Equations

In LEE, the mean flow is assumed to be given by other means, and the disturbances are small enough with respect to mean flow to permit linearization. In Ref. 106, the LEE approach was used to calculate both the source regime and the radiated field for a supersonic jet. The results for the source regime and the radiated sound agreed favorably with the corresponding measurements of Troutt and McLaughlin. Figure 5 shows a photograph of the pressure disturbance field associated with a supersonic jet at conditions corresponding to Troutt and McLaughlin's[107] experiment. The LEE approach was used to directly calculate both the source regime and the associated radiation. The figure shows a stable solution free from artificial reflection is obtained.

LEE approach is more general compared to approaches based on linear stability analysis. Because no particular assumption is made regarding the form of the disturbance, LEE can be used for direct prediction of both the flow and the acoustic disturbances. The simulation can also capture the simultaneous presence of several instability modes, and other modes including turbulent disturbances. Furthermore, LEE accounts for flow-divergence effects. However, unlike LES, the nonlinear effects, cannot be accounted for in LEE approach, and the mean flow must be given by another means. Because LEE focuses on calculating the disturbances alone, it is much less CPU demanding than LES, and is thus suitable for parametric studies and qualitative evaluation of new concepts.[115–117]

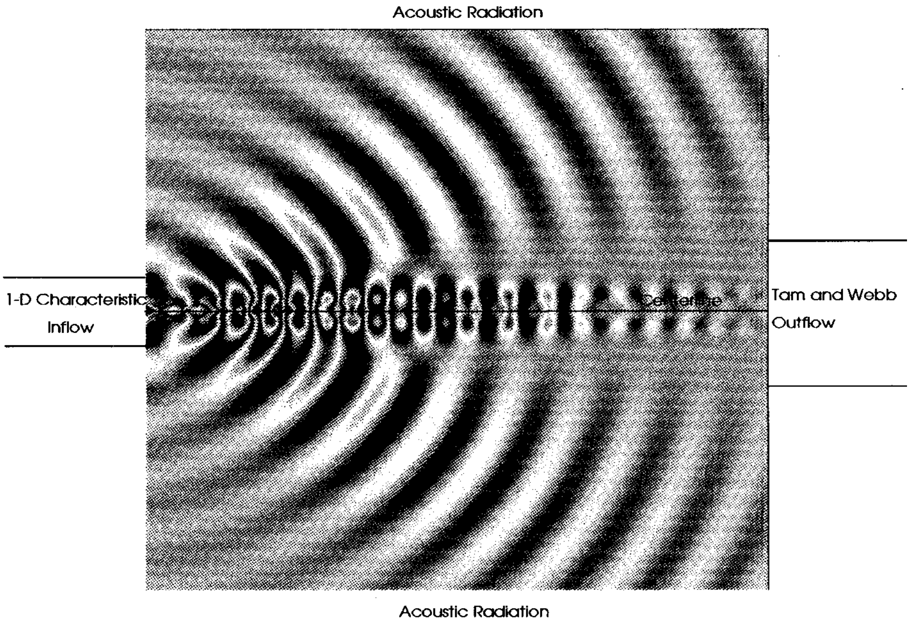

Fig. 5 Photograph of the disturbance field associated with a supersonic jet using LEE.

C. Integral Energy Approach

Experimental observations and numerical simulations indicate that turbulence in the noise-producing initial region of jets is dominated by coherent, wavelike structure.[70] The fluid motion can, therefore, be split into three kinds of motion: a time-averaged motion U, a periodic large-scale wavelike structure u', and a fine-scale random turbulence u'', and the pressure and density are similarly split. In the integral energy approach, the coherent, large-scale component is calculated while accounting for the effect of the fine-grained turbulence on it, as in LES. But rather than direct numerical computation, the following assumptions are first made. The coherent computation is taken to be composed of a finite number of wavelike frequency components in the form

$$u_i' = \sum_{m,n} \Re |A_{mn}(x)| \hat{u}_{i,mn}(r) \exp[i\psi_{mn}(x) - i\omega_m t + in\chi] \quad (11)$$

Here, ω is the frequency, n is the azimuthal wave number, and $\Re$ denotes the real part, and subscript m denotes the frequency component. The basic assumption here is that the Fourier coefficient representing the coherent component can be separated into an amplitude that varies with the downstream coordinate x and a radial shape function of the radial coordinate r at a given location along the shear layer or jet. The amplitude A is to be determined from nonlinear analysis; the radial profile $\hat{u}_{i,mn}(r)$ is taken as the eigenfunction given by the locally parallel linear stability theory. The phase angle ψ is governed by its own nonlinear evolution equation. With these shape assumptions, the energy equations are derived and integrated in the radial direction to yield a system of simultaneous ordinary differential equations describing the development of mean-flow momentum thickness, the turbulence kinetic energy, the energy and phase of each frequency component.[118,119]

In this integral energy approach, the nonlinear effects are accounted for by considering the interaction among various scales of motion (mean flow, turbulence, and various frequency components of the coherent structure). Because ordinary differential equations (rather than partial ones) are solved, the computational time is not an issue. The success of this approach depends mainly on the validity of the assumption regarding the profile for the coherent components. The approach seems to produce results consistent with observation, particularly in the initial region of developing shear layers, where most of the noise is generated.[120,121]

D. Morris et al.'s Approach

Morris et al.[122] proposed an approach that combines the benefits of LEE in being fast with that of LES in being nonlinear and viscous. The basic idea is to split the flow parameters into three components: mean flow quantity, large-scale fluctuating, and random fine-grained components. The mean flow is assumed to be given, as it is in LEE. This is substituted in the LES equations to obtain a nonlinear equation for the large-scale fluctuations. The essential advantage of Morris et al.'s scheme over the LEE approach is that nonlinear and viscous effects are accounted for. The drawback compared to LES is that the mean flow is considered independent of the calculated large-scale fluctuations. Shih et al.'s[123] attempt to address this issue by taking the mean flow to be initially given, as in Morris et al.,[122] but then corrected via an ordinary differential equation that brings in the effect of the resolved disturbance and the fine-grained turbulence. Efforts are still needed to assess the approximation involved, and the gain in CPU time compared with LES.

VI. Nozzle Flow and Acoustics

The unsteady flow inside the nozzle plays a key role in the jet exhaust noise. The far-field jet noise is essentially governed by the jet inflow, which is the nozzle exit flow. The latter is determined by the flow dynamics inside the nozzle. Furthermore, the acoustic field generated inside the nozzle directly radiates sound to the far field, and can also trigger a receptivity mechanism by which the jet shear-layer instability is amplified and radiates sound. As such, industry relies mainly on shaping the engine's interior geometry for jet noise suppression.

LES of realistic internal geometries is still not feasible, but simplified computations for idealized geometries were performed. Several investigators have considered radiation from a point source in a duct. Myers[124] calculated the radiation from a short duct assuming uniform mean flow, whereas Reichert and Biringen[125] considered the effect of lining. Effects of nonuniform mean flow were accounted for in Ref. 126 by developing inflow treatment that allow for entrainment to the duct to depict a real situation.

Usually jet plume is preceded by a mixer/ejector in which high-speed flow is mixed with a low-speed flow before it issues from the nozzle.[127–129] The problem is simplified in Ref. 130 to a simple mixing between two streams of different velocities separated by a cylindrical surface and bounded by solid walls. An LES code was then developed to study the acoustic and flow unsteadiness. Accurate treatment of the solid wall boundary condition was imposed to obtain proper reflection. Results show that a spatially growing mixing layer is formed starting at the separation point. Because of the mean flow gradient, the disturbances grow in amplitude. An outflow treatment was used that handles sharp mean-flow gradients supporting instability waves.

Plane wave fronts form and propagate in both the upstream and downstream directions and can leave the outflow and inflow boundaries without artificial reflections. While the simulation is for nominally perfectly expanded inner jet, a weak shock-cell structure is formed. This is attributed to the fluctuating nature of the disturbances, which causes the inner jet to alternate between underexpanded and overexpanded forming weak shock or expansion waves. The resulting unsteady disturbances can be taken as the inflow disturbances for jet plume computation. Thus, the engine geometry could be linked to the far-field radiated sound. But simulations of more realistic geometries are still needed.

VII. Concluding Remarks

Progress and difficulties associated with computational aeroacoustics of jet noise have been pointed out. Several algorithms can accurately handle the linear propagation of acoustic waves. Further developments and validations of schemes for handling nonlinear problems are still needed, particularly for multidimensional curvilinear grids involving shocks and acoustic waves. Careful selection and implementation of available boundary treatments can produce stable solutions free from spurious modes. However, further developments of inflow treatment are needed.

LES can simultaneously capture both the near flowfield and the radiated acoustic field, provided careful attention is given to boundary treatment and that a high-order scheme is used. Results indicate that the radiation pattern is dependent on the nature of the inflow disturbances. The only limitation of LES compared to DNS is that radiation caused by the smaller scales are not computed. But, if the LES resolution is such that the acoustically relevant scales are captured, the radiation from the unresolved scales should be negligible. Because of current computer capabilities, full three-dimensional LES simultaneously capturing both the near and far field is not practical. As such, one should limit LES to the near, nonlinear field, and use other faster techniques to compute the corresponding linear field.

For extending the nonlinear source regime to the far field, the finite difference solution of linearized Euler equations appears to be an accurate and a practical tool. This approach accounts for the simultaneous presence of various disturbance modes while accounting for effects of nonuniform mean flow. While the LEE solution is practically as accurate as LES in the acoustic-propagation regime, considerable saving in CPU time is achieved because the focus is on calculating the disturbance field apart from the mean flow. For the very far field, the no-derivative surface-integral approach, which relies on the pressure signal alone on the surface surrounding the sound source, is an appropriate tool. But it cannot account for the effects of nonuniform mean flow in the acoustic-propagation regime.

Several approximations to the prediction of the sound source were reviewed. While these methods are faster than LES, none of them is as accurate. Further studies are needed on the balance between approximations involved and the saving in computer time.

For CAA to payoff in propulsion systems, one has to be able to link engine design to the far-field generated sound. Unsteady flow in simple engine configurations has been calculated. Studies linking it to the far-field sound is only beginning. Furthermore, in these internal-noise calculations, the sound source is idealized by a simple monopole or simple mixing process. One has to turn to real components and calculate its associated disturbance field to be able to link it to the far-field noise.

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